Quality indicators for multi-objective optimization: performance assessment and algorithm design

Tutorial proposal for IEEE CEC 2023

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1 Introduction

For almost thirty years, multi-objective evolutionary algorithms (MOEAs) have been continuously used to approximate the solution of complex multi-objective optimization problems (MOPs) in different fields. Through these years, we have witnessed the birth of multiple MOEAs, each one having special design characteristics. In consequence, one important question needs to be answered: how do we evaluate the performance of MOEAs? The answer to this question is: Quality Indicators (QIs).

Before discussing what QIs are, we need to go to the basics. The solution of a MOP is a set of decision vectors (known as the Pareto set) whose images in the objective space represent the best-possible trade-offs among the objective functions that shape the Pareto front [1]. The main goal of MOEAs is to produce a finite approximation of the Pareto[1] front having the following three properties [2]: 1) convergence: the candidate solutions should be as close as possible to the Pareto front, 2) spread: the candidate solutions should cover the whole Pareto front, and 3) uniformity: the candidate solutions should be uniformly distributed along the Pareto front. In the first years of evolutionary multi-objective optimization (EMO), visual comparisons of the Pareto front approximations were performed to decide which MOEA was the best. This was possible because the Pareto front approximations were embedded into two- or even three-dimensional objective spaces. However, as long as MOEAs acquired more power to tackle MOPs with more than three objective functions (i.e., the so-called many-objective optimization problems (MaOPs)), a visual comparison was difficult or even not possible. In consequence, a need for quantitative mechanisms to compare MOEAs was mandatory.

[1]We denote a Pareto front approximation as $A = \{\vec{a}_1, \ldots, \vec{a}_N\}$, where $\vec{a}_j \in \mathbb{R}^m$ is an objective vector and m is the number of objective functions.

In 1999, Van Veldhuizen [3] settled down the basis of QIs to quantitatively compare MOEAs. A k-ary QI (I) is a set function that receives as input k Pareto front approximations and outputs a single real value: $I : A_1 \times \cdots \times A_k \rightarrow \mathbb{R}$, where A_i is a Pareto front approximation. Later, in 2003, Zitzler *et al.*[4] mathematically formalize the concept of a QI and they established the first guidelines to compare the Pareto front approximations generated by MOEAs. Since then, several researchers around the world have made efforts to increase the knowledge on QIs and performance assessment of MOEAs [5, 6, 7, 8, 9, 10, 11]. Currently, there are several QIs that measures the three principal aspects of a Pareto front approximation, i.e., convergence, spread, and uniformity [5].

The main goal of a QI is to evaluate the quality of a Pareto front approximation. In other words, it processes all the objective vectors of an approximation set and generates a numerical value that reflects the quality. In consequence, one could take advantage of a QI to guide the evolutionary process of a MOEA, giving rise to the so-called indicator-based MOEAs (IB-MOEAs) [12]. The underlying idea of an IB-MOEA is to approximate the indicator-based subset selection problem such that the best-contributing solutions to the overall quality of the Pareto front approximation survive the selection process. The final Pareto front approximation produced by an IB-MOEA exhibits the specific preferences of its baseline QI. For example, the $\mathcal{S}$-Metric Selection Evolutionary Multi-Objective Algorithm (SMS-EMOA) [13] shows a preference to objective vectors near the Pareto front's knee and in the boundary since it is based on the hypervolume indicator (HV) [14]. In contrast, the $R2$-based EMOA [15], that uses the $R2$ indicator [9], produces uniformly distributed Pareto front approximations when the associated manifold is concave and correlated with the shape of an m-dimensional simplex. Regardless of the baseline QI utilized by an IB-MOEA, these metaheuristics have been a promising way to increase the selection pressure when tackling MaOPs and, thus, they have attracted the attention of the EMO community in the 15 years [12].

Currently, the research on QIs and IB-MOEAs has advanced significantly which is reflected in the amount of available papers in the specialized literature. Unfortunately, this research area has been developed by a few research groups around the world and it is still missing a good difussion of the currente developments and open research areas. Hence, we propose a tutorial that promotes the difussion of the current advances on QIs and IB-MOEAs such that the community has an easy access to the information and knowledge. Furthermore, the tutorial aims provide both theoretical and practical aspects of QIs and IB-MOEAs with the intention of attracting more scientists to this research area and allow the practitioners to use the current performance comparison techniques and algorithms.

2 Tutorial description

This tutorial aims to introduce researchers, students, and practitioners to the current state-of-the-art on QIs for multi-objective optimization. The two main objectives of the tutorial are 1) the utilization of QIs to quantitatively evaluate the performance of MOEAs, and 2) designing new MOEAs using QIs as the backbone of selection mechanisms. Due to the nature of the topics covered in the tutorial, we expect 4 hours of duration. In addition to the theoretical concepts, we will provide practical activities to improve the understanding of the theory, and, in general, to provide a high-quality and exciting tutorial. The tutorial is projected at an Introductory level. The content of the tutorial is the following.

1. A brief introduction to evolutionary multi-objective optimization.
2. Quality indicators: what are they?
3. Theoretical foundation
4. QIs for performance assessment
   (a) A classification of QIs
      i. Convergence QIs
      ii. Diversity QIs: Spread and Uniformity
      iii. Convergence-Diversity QIs
      iv. Cardinality QIs
      v. Combined QIs
   (b) Performance comparison of MOEAs: techniques and restrictions
   (c) Future trends in performance assessment
   (d) Play time: assessing MOEAs with QIs
5. QIs for algorithm design
   (a) Theoretical basis: subset selection problem
   (b) Single-Indicator-based MOEAs: state-of-the-art approaches
   (c) Multi-Indicator-based MOEAs: a new research field
   (d) Indicator-based Local Search Techniques
   (e) Future trends in designing IB-MOEAs and MIB-MOEAs
   (f) Play time: designing a new IB-MOEA
6. Open research areas
7. Conclusions

The tutorial is divided in 7 sections, where the first 4 sections will be covered in the first 2 hours of the tutorial and Sections 5 to 7 will be covered in the last 2 hours. First, it is necessary to introduce the general concepts of multi-objective optimization and how to tackle MOPs with MOEAs. Then, the general concept of QI is presented in Section 2, giving the theoretical basis of QIs and how to compare MOEAs in Section 3. Section 4, is the most important in the first half of the tutorial. In this section, we will review several state-of-the-art QIs, emphasizing how to use all the QIs for performance assessment. To conclude this first part of the tutorial, we will provide the audience a software framework for multi-objective optimization where they willl execute different MOEAs to perform a comparison analysis using QIs. Moreover, the attendants will have the chance to implement a QI in the framework to use it in the comparison. In the second half of the tutorial, we will focus on the utilization of QIs to create MOEAs. We will review several state-of-the-art IB-MOEAs, MIB-MOEAs, and indicator-based local search techniques. In this section, we will also make use of the framework such that the audience implements a new IB-MOEA besides using some other IB-MOEAs. To finalize the tutorial, we will discuss some future research directions for QIs and IB-MOEAs and we will sketch our final conclusions.

3 Team members

This tutorial will be provided by 3 young researchers from Mexico and China. In the last years, the three members, Dr. Jesús Guillermo Falón-Cardona, Dr. Ke Shang, and Dr. Víctor Adrián Hernández Sosa, have been very active in the field of IB-MOEAs. Their research has covered topics such as theoretical aspects of QIs and IB-MOEAs, the design of new IB-MOEAs and MIB-MOEAs, the utilization of indicator-based local search techniques, and the proposal of new QIs for convergence and diversity. Moreover, the three team members have experience publishing their research at prestigious international conferences such as IEEE CEC, ACM GECCO, PPSN, EMO, and, IEEE SSCI, as well as in international journals such as IEEE Transactions on Evolutionary Computation, Evolutionary Computation Journal, ACM Computing Surveys, Swarm and Evolutionary Computation, Soft Computing, Memetic Computing, among other important journals. Another important aspect of a successful tutorial is the teaching experience of the three researchers in undergraduate and graduate programs. Hence, we are confident in providing a high-quality tutorial. A full description of the team members is provided in the following.

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